PERFORMANCE OF A SINGLE STACK COLD PLATE WITH HEAT LOSSES

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Abstract A single stack cold plate used for the cooling of electronic components is analysed under steady state conditions using finite element method where Galerkin's weighted residual method is employed. A simple one-dimensional fin theory is applied to the descretised elements during the analysis. The formulation of the analysis is more general and takes into account the heat losses from the top and bottom surfaces of the stack. First, a single unit cell without heat losses is analysed whose results compare well with those available in the literature. Then the analyses of the assembly of several unit cells with different heat losses are carried out. These results show that the single unit cell can be considered as the representative of the stack only when there are no heat losses.

Keywords: Single stack cold plate, Finite element method, Fin theory, Heat losses.

INTRODUCTION

Cold plates are used to cool electronic components mounted on the printed circuit board. A cold plate consists of an array of rectangular fins, attached between two exterior plates. Fluid passes through the spaces between the fins, to help increase the rate of heat transfer from the fin. A cold plate is called a single stack cold plate if there is no splitter plate between the two exterior plates; double stack if there is one splitter plate and triple stack if there are two splitter plates.

A stack is a combination of fins connected together. Thus, the whole stack is made of repeating arrays of fins. The objective of this paper is to generate new and additional data, which will be helpful in the design of cold plates used for cooling of electronic systems. This is achieved by a generalized formulation of the analysis of a cold plate, using dimensionless parameters to replace the dimensional parameters used in the earlier investigations. By non-dimensionalizing the governing equation, the analysis is not restricted to a particular set of geometry of a stack. The analysis starts by taking a single unit cell of the cold plates for the purpose of verifying results with those from Pieper and Kraus [1998]. In addition to that, several unit cells assembled together vertically are analysed using the finite element method with and without heat losses at the top and bottom of the stack as it happens in reality. Results of such analysis will determine if a single unit cell analysis is adequate in representing the performance of a stack under all operating conditions.

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THE GOVERNING EQUATION AND FINITE ELEMENT FORMULATION

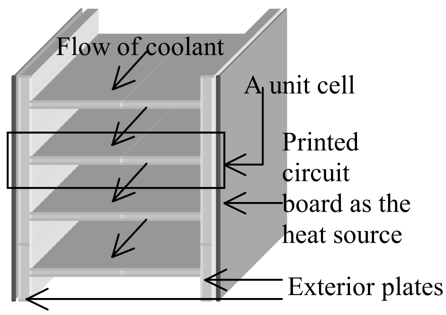

Fig. 1 A single stack cold plate

A stack is considered as a combination of fins connected together. A simple one-dimensional fintheory is therefore applied to the stack under investigation.

The governing equation for a steady state, one-dimensional fin, with conduction and forced convection is given as follows:

$$kA\frac{d^2T}{dx^2} - hP(T - T_\infty) = 0 \qquad (1)$$

where
A = cross sectional area of the fin perpendicular to the direction of conduction, m^2
h = heat transfer coefficient, W/m^2 °C
k = thermal conductivity of the fin material, W/m °C
P = perimeter of the fin where convection takes place,m
T = temperature of the fin at a given location, °C
T_∞ = ambient temperature, °C
x = distance measured from the base of a fin, m

As a generalisation, the present analysis will be carried out using dimensionless parameters. This is carried out after a transformation of the field variables to their corresponding dimensionless parameters as follows:

$$\tilde{Q} = \frac{\text{Heat loading on the right exterior plate}}{\text{Heat loading on the left exterior plate}} = \frac{Q_{TR}}{Q_{TL}} \qquad (2)$$

$$X = \frac{x}{B}\text{, where } B = \text{reference dimension, m} \qquad (3)$$

$$\theta = \frac{kA(T - T_\infty)}{Q_{TL}B} \qquad (4)$$

$$M = \frac{hPB^2}{kA} \qquad (5)$$

Thus, equation (1) becomes

$$\frac{d^2\theta}{dX^2} - M\theta = 0 \qquad (6)$$

M is the governing parameter in the analysis that takes into account the variation in h (free convection, mixed convection and forced convection including the developing flow), the variation in k (different fin materials), geometric factor (A/P ratio of the flow passage) and finally the distance between theExterior plates. Thus the introduction of the parameter M does not restrict the present analysis to a particular set of geometry.

The variation of temperature along the fin is assumed to be linear as $\theta = [N]\{\theta\}$
(7)
where

$$[N] = [1 - X \quad X] \qquad (8)$$

$$\{\theta\} = \begin{Bmatrix} \theta_i \\ \theta_j \end{Bmatrix} \qquad (9)$$

θ_i, θ_j = dimensionless temperature at nodes i and j respectively.

By using Galerkin's method, as explained in Segerlind [1984] and Lewis et. al. [1996], the finite element formulation of equation (6) is obtained as

$$[K]\{\theta\} = \{0\} \qquad (10)$$

where

$$[K] = \begin{bmatrix} 1 + \frac{M}{3} & -1 + \frac{M}{6} \\ -1 + \frac{M}{6} & 1 + \frac{M}{3} \end{bmatrix} \qquad (11)$$

The above theory is applied to fin array and single stack cold plate. The details of the assembly of the element matrices for each case are given in the following section.

FIN ARRAY

In order to verify the present approach, we consider the case of a fin array for which solutions are available. Mikhailov and Ozisik [1981] modeled a fin array using a linear combination of two fundamental solutions to the governing differential equation for the one-dimensional steady state problem. However, in the present analysis each fin can be considered to have more than one element. Thus the present finite element analysis is more general and can be used for longer fins as well.

Considering four elements of the fin array (as shown in Fig. 2a), the element matrix for element 1 is written as

$$\begin{bmatrix} 1 + \frac{M_1}{3} & -1 + \frac{M_1}{6} \\ -1 + \frac{M_1}{6} & 1 + \frac{M_1}{3} \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad (12)$$

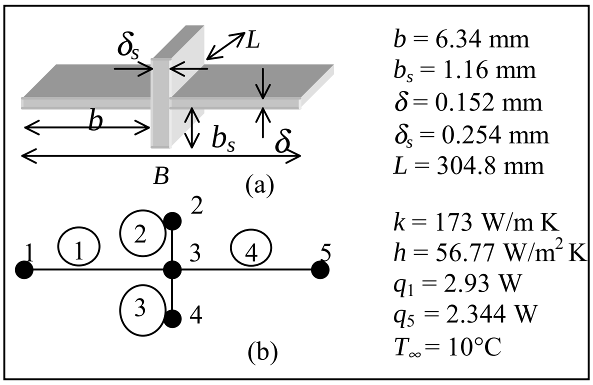

Fig. 2a Geometry and dimensions of a fin array [Mikhailov and Ozisik, 1981]
Fig. 2b Finite element representation of a fin array

where

$$M_1 = \frac{hP_1B_1^2}{kA_1} \qquad (5a)$$

P_1 = perimeter of fin 1, m
B_1 = width of fin 1, m
A_1 = cross sectional area of fin 1, m^2

Similarly, the element matrices for elements 2, 3 and 4 are as follows:

$$\begin{bmatrix} 1+\frac{M_2}{3} & -1+\frac{M_2}{6} \\ -1+\frac{M_2}{6} & 1+\frac{M_2}{3} \end{bmatrix} \begin{Bmatrix} \theta_2 \\ \theta_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (13)$$

$$\begin{bmatrix} 1+\frac{M_3}{3} & -1+\frac{M_3}{6} \\ -1+\frac{M_3}{6} & 1+\frac{M_3}{3} \end{bmatrix} \begin{Bmatrix} \theta_3 \\ \theta_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (14)$$

$$\begin{bmatrix} 1+\frac{M_4}{3} & -1+\frac{M_4}{6} \\ -1+\frac{M_4}{6} & 1+\frac{M_4}{3} \end{bmatrix} \begin{Bmatrix} \theta_3 \\ \theta_5 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (15)$$

where M_2, M_3 and M_4 are defined as in equation 5(a) for fins 2, 3 and 4 respectively.

The global matrix is an assembly of the element matrices. After incorporating the heat loadings at node 1 (q_1) and node 5 (q_5), the global matrix is given as in equation (16).

Table 1 A comparison of steady state excess temperature

Node	Excess temperature in °C (Mikhailov and Ozisik, 1981)	θ from present analysis	Excess temperature in °C from present analysis
1	11.20	2.36	11.16
2	9.77	2.06	9.74
3	9.78	2.06	9.76
4	9.77	2.06	9.74
5	10.76	2.27	10.72

Table 1 shows the results of Mikhailov and Ozisik [1981]. Present analysis considers eight elements. Mikhailov and Ozisik [1981] used excess temperature, which is equal to the actual nodal temperature minus the ambient temperature. These excess temperatures are converted to the non-dimensional values and are shown in Table 1 for comparison.

It is clear from Table 1 that there is a close agreement between the results of Mikhailov and Ozisik [1981] and the present analysis. This confirms that the present approach is valid for the assembly of fins.

SINGLE STACK COLD PLATE

The analysis of a single stack cold plate is presented. A single unit cell, being the repeating segments of the stack, is considered first. The analysis of this single unit cell is carried out under the same operating conditions as given by Pieper and Kraus [1998]. Results from the present analysis are then compared to those of Pieper and Kraus [1998] for the purpose of verification. Next, different number of unit cells are considered and analysed after having assembled them together vertically with and without heat losses from the top and the bottom of the assembled unit. Results from such analysis are compared to those of Pieper and Kraus [1998] and conclusions are drawn.

Analysis of a single unit cell

Pieper and Kraus [1998] analysed the single stack cold plate under the steady state conditions without heat losses from the top and the bottom surface of the stack. The geometry of a single unit cell is given in Fig. 3a. Fig. 3b shows a unit cell, which is discretised into six one-dimensional fin elements.

It may be mentioned here that the heat loading per unit cell on either side of the exterior plates are divided equally at the nodes of the elements considered. The heat loading for the whole stack at the left exterior plate is Q_{TL}, while the same is represented by Q_{TR} for the right exterior plate. The heat loadings are non-dimensionalised with reference to the heat load on the left exterior plate, as given in equation (2). The reference dimension in equation (3) is the total width of the stack. NUC is used as the abbreviation for the number of unit cells. NUC = 44.4 and M = 0.55 for the case considered in Pieper and Kraus [1998]. The results obtained can then be compared to those of Pieper and Kraus [1998], who presented the results of excess temperatures for a single stack cold plate for different heat loadings on the right exterior plate and a constant heat loading on the left exterior plate.

$$\begin{bmatrix} \left(1+\frac{M_1}{3}\right) & 0 & \left(-1+\frac{M_1}{6}\right) & 0 & 0 \\ 0 & \left(1+\frac{M_2}{3}\right) & \left(-1+\frac{M_2}{6}\right) & 0 & 0 \\ \left(-1+\frac{M_1}{6}\right) & \left(-1+\frac{M_2}{6}\right) & \left(4+\frac{M_1}{3}+\frac{M_2}{3}+\frac{M_3}{3}+\frac{M_4}{3}\right) & \left(-1+\frac{M_3}{6}\right) & \left(-1+\frac{M_4}{6}\right) \\ 0 & 0 & \left(-1+\frac{M_3}{6}\right) & \left(1+\frac{M_3}{3}\right) & 0 \\ 0 & 0 & \left(-1+\frac{M_4}{6}\right) & 0 & \left(1+\frac{M_4}{3}\right) \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \end{Bmatrix} = \begin{Bmatrix} q_1 \\ 0 \\ 0 \\ 0 \\ q_5 \end{Bmatrix} \quad (16)$$

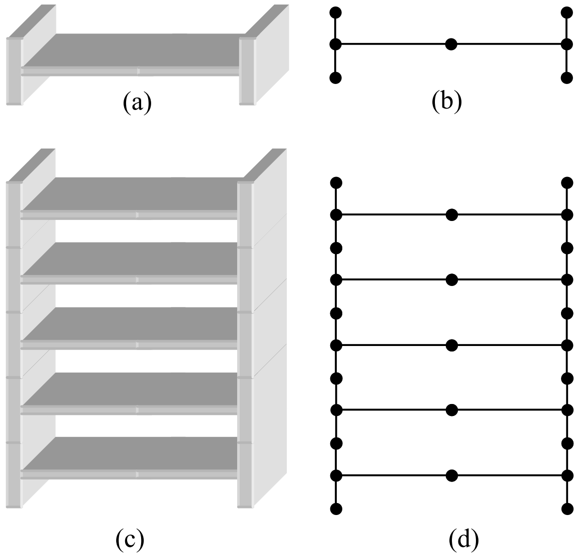

Fig. 3
a Geometry of a unit cell of a single stack cold plate
b Finite element representation of a unit cell of a single stack cold plate
c Geometry of the assembly of 5 unit cells of a single stack cold plate
d Finite element representation of the assembly of 5 unit cells of a single stack cold plate

Fig. 4 shows the maximum dimensionless temperatures on the left (θ_l) and right (θ_r) exterior plates. The highest value among the three nodes on each exterior plate is taken as the representative value. The results of Pieper and Kraus [1998] are also shown in Fig 4. There is a good agreement of results between the present analysis and the results reported by Pieper and Kraus [1998]. It shows that the present analysis applied to the single stack cold plate is correct.

Analysis of stack with varying NUC

Pieper and Kraus [1998] assumed that if the cold plate is made up of more than 20 repeating segments, a single segment is adequate for the purpose of analysis. Moreover, they neglect the edge effect of the stack. In the present analysis, several unit cells of a single stack cold plate are assembled in the vertical direction as in Fig 3c and analysed for the same operating conditions. The purpose of analysing the assembled unit cells is to determine whether the assumption made by Pieper and Kraus [1998] regarding the analysis based on a single cell is correct. Furthermore, the edge effect is brought into account by allowing heat losses from the top and bottom surface of the stack.

Analyses of the assembled unit cells of varying number with and without heat losses are carried out. The heat loss is defined as a percentage of the total heat loading at the left exterior plate Q_{TL}. Since Q_{TL} increases proportionally with NUC, the heat loss also increases

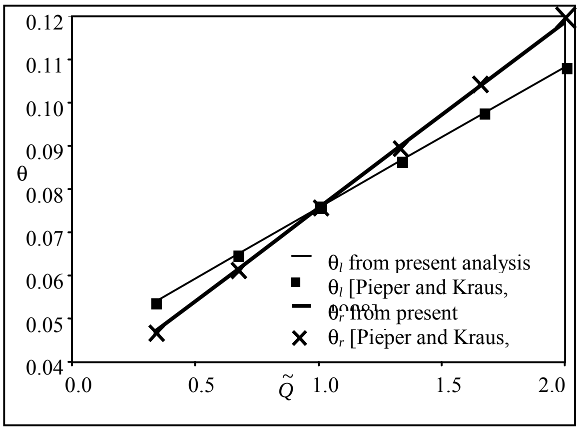

Fig. 4 Dimensionless temperature versus heat loading for a single cell analysis.

accordingly. This also implies that a heat loss of 10% for NUC = 20 is twenty times larger than a heat loss of 10% for NUC = 1. This definition of heat loss also implies that for a fixed NUC, a heat loss of 10% is the same irrespective of $\tilde{Q}$.

Fig. 5 shows different curves of the temperature distribution at the left exterior plate for NUC = 1, 5, 20, 50 and 100 with heat loss (HL) from both ends = 0, 0.1 and 0.2 and $\tilde{Q}$ = 1. In Fig. 5, notations like $(HL0)_1$, $(HL1)_5$, … $(HL2)_1$, $(HL2)_5$ etc are used where HL stands for the heat loss, the number after HL, (0, 1, 2) represents zero loss, 0.1 (10%) loss and 0.2 (20%) loss respectively and the subscripts refer to the number of unit cells (NUC) considered for the analysis. It is observed from the figure that when there is no heat loss from both the ends, the temperature distribution is uniform throughout irrespective of NUC as shown by the curves $(HL0)_1$, $(HL0)_5$, $(HL0)_{20}$, $(HL0)_{50}$ and $(HL0)_{100}$. This justifies the assumption made by Pieper and Kraus [1998] that a single unit cell is a

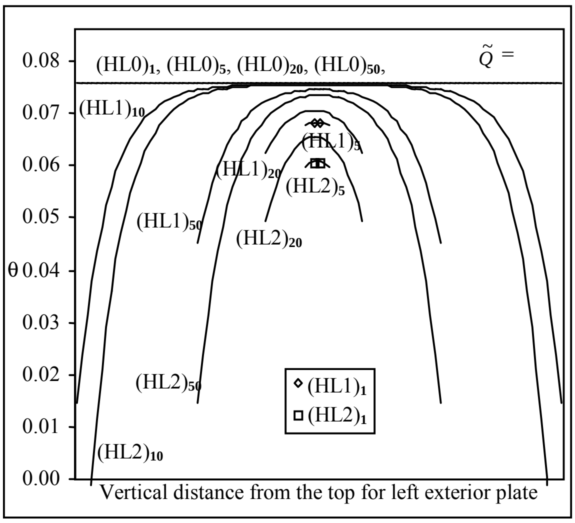

Fig. 5 Temperature distribution at the left exterior plate for various NUC.

representative of the stack and is adequate for the purpose of analysis if there is no heat loss. When the heat loss equals 0.1, the maximum temperature θ_l obtained for NUC = 1 drops as compared to the earlier case (zero heat loss) as can be observed from the curve $(HL1)_1$. Results obtained for NUC = 5 and HL = 0.1 show that there is not much difference in θ_l as compared to that obtained for NUC = 1 and HL = 0.1 (curves $(HL1)_1$ and $(HL1)_5$). The same trend is observed when calculations are carried out for HL = 0.2. When NUC = 20, the value of θ_l is higher than that obtained for NUC = 1 or 5 when both heat losses are considered. Similar behaviour is observed when NUC is increased to 50. The lower temperatures at the ends and the symmetrical temperature distributions when the heat loss is considered are clearly shown by the different curves in Fig. 5. However, analyses with NUC = 100 and HL = 0.1 and 0.2 reveal that the temperatures of the near middle cells are closer to those obtained for the case with no heat loss as can be seen from the curves $(HL1)_{100}$, $(HL2)_{100}$, $(HL0)_1$, $(HL0)_5$, …$(HL0)_{100}$. Furthermore, the curves $(HL1)_{100}$ and $(HL2)_{100}$ representing the temperature variation along the left exterior plate for different heat losses are very close to each other near the middle cells and differ at other locations. These results are not far from expectation. From the above analysis, it can be concluded that for large NUC (≥100) with heat losses taking place, the analysis with a single unit cell at the middle without heat loss is adequate to get θ_l, which will be helpful to determine whether the maximum temperature limit θ_{max} has been achieved or not. The above analysis also shows that a single unit cell analysis with heat loss does not represent the conditions of the middle cell of a stack having number of repeating segments of unit cell less than 100.

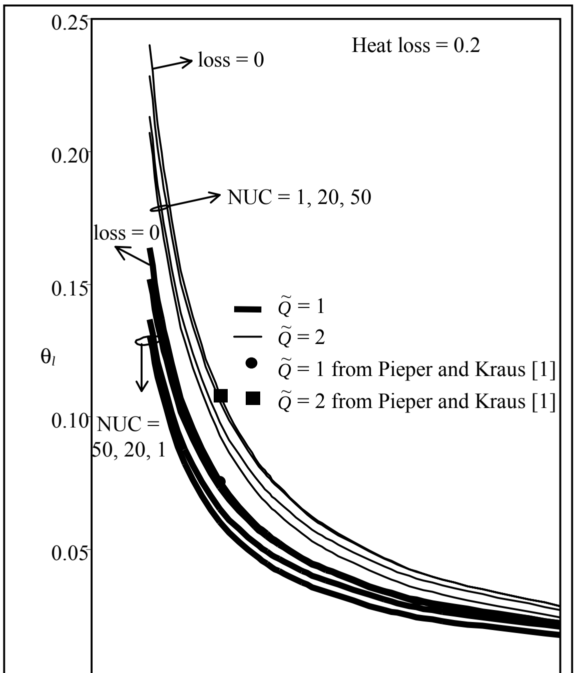

Fig. 6 θ_l versus *M*

All the above results and discussions were limited to a particular value of the dimensionless parameter *M* = 0.55 for a single stack cold plate, which corresponds to the case of Pieper and Kraus [1998]. In order to present a generalized behaviour of the cold plate, analyses for different values of *M* are carried out with and without heat losses from the top and bottom surfaces of the stack. These analyses have generated new and additional data, which will be helpful in the design of the cold plate used for the cooling of the electronic systems. The results are plotted in terms of the maximum dimensionless temperature on the left exterior plate θ_l for different values of *M* and for two different values of heat loading $\tilde{Q}$ when the heat loss = 0.2 as shown in Fig. 6. The effect of the variation of NUC is also shown in the figure. Calculations are also carried out for no heat loss and the results are plotted in the same figure. Thus Fig. 6 represents a generalized curve applicable for any single stack cold plate for $\tilde{Q}$ = 1 & 2 with a heat loss = 0.2. In general, θ_l for $\tilde{Q}$ = 2 is higher than that for $\tilde{Q}$ = 1 for all the values of 0.25 < *M* < 2. For lower values of *M*, the difference between the values of θ_l for the two values of $\tilde{Q}$ is large but this difference reduces for *M* > 0.5. A single point on each curve, corresponding to zero heat loss for the two heat loadings, refers to the result presented by Pieper and Kraus [1].

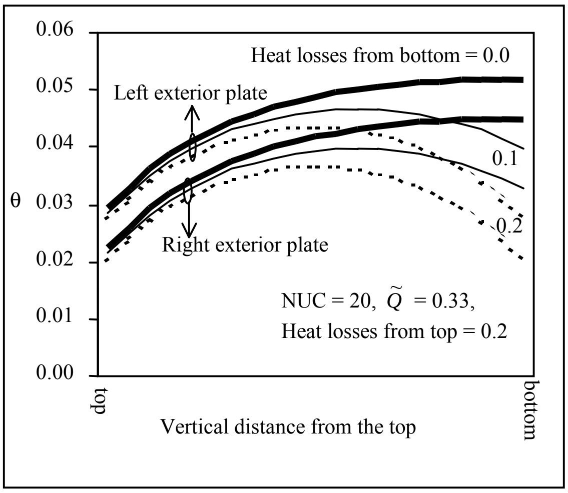

Fig. 7 Temperature distribution at the left and right exterior plate

It is also noticed from Fig. 6 that for $\tilde{Q}$ = 2, NUC = 50 and *M* > 0.75, θ_l coincide with those calculated for zero heat loss case. Similar observation is noticed from the lower curves for $\tilde{Q}$ = 1 but beyond a slightly higher value of *M* as compared to that established for $\tilde{Q}$ = 2. This suggests that for values of *M* ≥ 0.75 and even with a heat loss of 0.2, the results can be obtained from the analysis of a single unit cell without heat loss when NUC exceeds 50.

It is also possible to calculate the temperature distribution at the left and right exterior plates if the heat loss from the top and bottom of the assembled unit cells are different as happens in actual situation. Fig. 7 shows such results for the case where NUC = 20, $\tilde{Q} = 0.33$, heat loss from the top is 0.2, while heat loss from the bottom is varied from 0 to 0.2. When the heat loss at the top is larger than at the bottom, temperature drop at the top of the cold plate is more than the bottom. When the heat losses at both ends are the same, temperature distribution is symmetric about the middle of the stack.

CONCLUSIONS

From the above analysis of the single stack cold plate, the following conclusions are drawn:

1. When there are no heat losses from the top and bottom surfaces of the stack, the analysis of a single unit cell is adequate to determine the performance of the single stack cold plate.

2. For M ≤ 0.55, anlaysis of the whole stack should be carried out when heat losses are to be considered otherwise the performance prediction will be different. However, it is concluded that for a stack of NUC ≥ 100 and heat losses being considered, the maximum temperature in this case is the same as that obtained from the analysis of a single unit cell without heat losses under the same heat loadings at the exterior plates.

3. For higher values of M (>0.75) and heat losses being considered, the single unit cell analysis is sufficient to get the maximum temperature even when NUC ≥ 50.

4. The expected variation in the results in terms of the temperature distribution along the left and right exterior plates for heat losses from the top surface being different from the bottom surface for a particular heat loading and a particular value of *M* are predicted well.

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